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Experimental Study for Design of Computational Learning Support to Enhance Problem Posing

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Abstract: In the learning of problem posing, even though it is difficult for learners to generate diverse problems by associating and combining various situations expressed in problem texts and mathematical structures of solutions, they must do so. To design supporting methods for expanding the variety of problems, it is crucial to understand the types of problems that learners have difficulty in posing, and to draw crucial elements to facilitate diverse problem posing. We conducted an experimental investigation to obtain the empirical data of the variety of mathematical word problems posed by novice participants. The results indicated that the participants posed only a few problems that had situations identical to and solutions different from given base problems. It was also revealed that the problems constructed by the participants based on their own ideas were relatively simple and inappropriate. These results suggest that idea generation support for constructing solution structures of problems is effective in expanding the variety of problem posing.

Keywords: Mathematical learning, problem posing, production task, computational learning support strategies

Introduction

In general mathematical learning, students solve many problems provided by teachers or textbooks. But besides problem solving, problem posing has also been identified as an important activity in mathematics education [5]. In fact, recent studies in educational systems have adopted problem posing as a learning task or have approached to problem posing support (e.g., [3, 4, 6]).

Problem posing is a production task that requires idea generation. In problem posing, learners have to generate new ideas because new problems cannot be composed only from given information in the task. In the learning of problem posing, it is critical for learners to pose diverse problems, but it is also difficult for them. The problems generated by novice learners lack diversity [1]. Thus, supporting strategies must be designed to promote diverse problem posing for learners, and that need a method to describe the variety of ideas in it. In this study, the variety of problems is described in the dimensions of two attributes: contextual settings expressed in problem texts (*situations*) and mathematical structures in problem solutions (*solutions*). Analogy research in cognitive science focuses on these two attributes of the surface and the structural features of problems as key factors in human problem posing (e.g., [2]). They are also essential in problem posing.

To design supporting methods for the learning of problem posing, it should be an indispensable approach to understand problem posing from the viewpoint of the variety. This study experimentally investigated problem posing to obtain empirical data of the variety of problems posed by novices. We then discussed the difficulties in the problem posing and crucial elements in computational supporting approaches.

1. Experimental Investigation

Our investigation was conducted in a cognitive science class that included topics related to creativity. Undergraduates participated in two problem posing tasks. In each task, they were asked to pose as many new problems as possible from given problems as bases, and to write their texts and solutions on provided sheets in 20 minutes. In Task 1, a word problem solved by a unitary equation was presented as a base, and Task 2 used another problem solved by simultaneous equations. Prior to starting the tasks, the participants were strongly encouraged to pose diverse and unique problems.
The problems posed by the participants were classified into four categories shown in Figure 1, which denoting similarities in the two dimensions, by judging whether their situations and solutions were identical to or different from the bases.

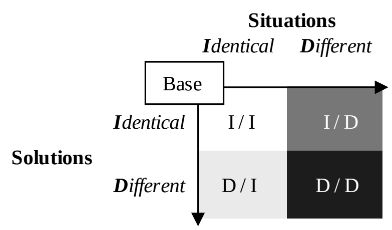

Figure 1. Categories for evaluating the variety of problems

2. Results and Discussion

Seventy six participants posed 146 problems in Task 1, and 73 participants posed 151 in Task 2. In data analysis, problems that were not solved by equations (29 in Task 1 and 39 in Task 2) and those that were unsolvable due to insufficient problem conditions (five in Task 1 and one in Task 2) were excluded. The following results include 112 problems in Task 1 and 111 in Task 2.

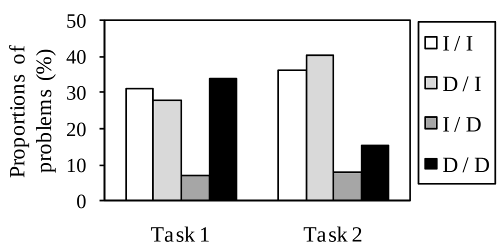

Figure 2. Proportions of posed problems in each category

Figure 2 indicates the proportions of the posed problems in each category in Tasks 1 and 2. Many problems in I / I and D / I were posed, but problems in I / D were few. The results of the two tasks were different in problems in D / D; many were posed in Task 1 and few in Task 2. These results revealed that posing problems in I / I and D / I was easy for the participants. They also posed problems in D / D to some extent, although the extent depended on the problem domains. On the other hand, problems in I / D were few in both of the two tasks. These facts confirm that it is difficult or unfamiliar for novices to pose new problems by only altering the solutions while controlling the situations.

To pose problems in I / D and D / D, the participants had to construct novel solutions by altering solutions of the bases in some ways. To do so, they partially altered the solutions of the bases by adding or removing operations, or they overall altered the solutions by constructing solution structures entirely different from the bases. Actually, 80.4 % of the problems in I /D and D / D were posed by the overall alteration in Task 1, and 65.4 % in Task 2. The participants may have tried to generate their own ideas entirely different from the bases because they had been encouraged to generate unique ideas. Thus, we have to examine the features of the solutions constructed based on the participants' own ideas from the aspect of solution complexity. Figure 3 indicates the proportions of the overall altered problems whose operations in the solutions increased more than bases in the tasks, whose operations were the same as the bases and whose operations decreased more than the bases. As the figure shows, half of the problems decreased the numbers of operations; that is, many problems were simpler than the bases. Particularly in Task 1, the operations for the absolute terms increased and those for the coefficients decreased, indicating that the participant problems had many numeric values but few descriptions related to their answers. Such problems were not excellent or appropriate as problems solved by setting up equations by extracting mathematical relations about answers. The participant problems varied from the bases but didn't necessarily successfully generate good ideas, although good responses in problem posing do not always generate problems with many solution steps. They may not have had sufficient ideas to adequately alter the solutions in problem posing. Based on these findings, to expand the variety of appropriate problem posing of novice learners, we need to design methods to support learners' idea generation for altering solutions.

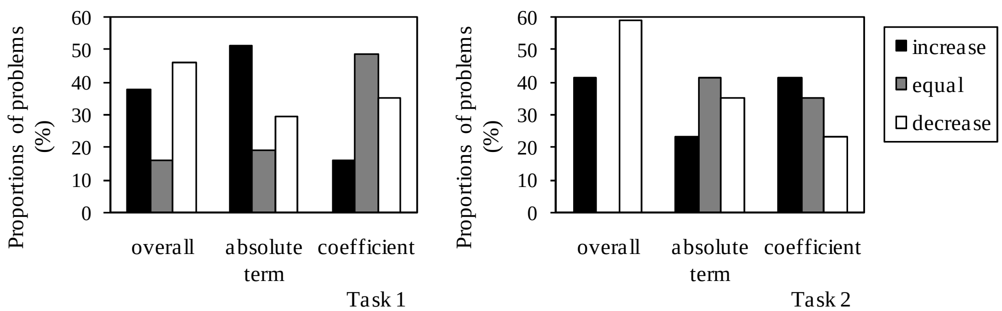

Figure 3. Proportions of overall altered problems increasing or decreasing operations

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