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A Learning Environment for Solution-based Problem-Posing in Multi-Digit Subtraction

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Abstract: In this paper, we describe developing and evaluation of a learning environment for *Solution-based Problem-Posing* in the case of multi-digit subtraction. One of the ways to learn by problem-posing is to make learners pose problems which can be solved by a solution. We call the type of problem posing *Solution-based Problem-osing*. The *Solution-based Problem-Posing* helps the learners understand the solution. In the case of multi-digit subtraction, however, it is difficult to provide the solutions directly because the solutions in the multi-digit subtraction are complex for learners who are elementary school students when they learn them. We suggest a method they are required to solve problems to be conscious the solution of the problems and then pose problems based on the solution.

Keywords: Interactive Learning Environment, Problem-Posing, Multi-Digit Subtraction

Introduction

In this paper, we describe a development an Interactive Learning Environment (ILE) for multi-digit subtraction and report an evaluation in a lecture of a class in an elementary school experimentally. In learning for multi-digit subtraction, even if learners can solve a problem, they don't always understand a meaning of a solution of the problem and condition of the solution [1]. Several researchers suggested that it is effective to master solution that a learner poses problems which can be solved by the solution [2] [3]. We called the type of problem-posing *Solution-based Problem-Posing.* We have already developed some ILEs for arithmetical word problems solved by an addition or a subtraction [4]. Kojima and his colleagues have also developed ILEs for posing arithmetical word problems [5]. Instead of mastering solution, their ILEs intend to diversify learners' problem posing. Solution in arithmetical word problems can be shown in terms because its structure is relatively simple compared with problem structure, but solution in multi-digit subtraction is complex. Besides, it is difficult for early elementary school children to understand the provided explanation in terms. So, in multi-digit subtraction, it is difficult to pose problems with a same method as *Solution-based Problem-Posing* for arithmetical word problems. We suggest a method that learners solve and consider a problem and pose the problem which can be solved by the used solution.

1. Problem-posing for multi-digit subtraction

1.1 Solution-based Problem-Posing

We proposed a problem-posing method composed of the two steps following: First, learners solve a problem. Second, they pose a problem which has same solution of the solved problem. This method makes it possible for them to pose a problem based on solution without explanation about complex solution method of multi-digit subtraction. This method is effective in multi-digit subtraction because the solution is complex to be explained by sentences and elementary school students are too young to understand it.

1.2 Categorizing Structure of Problems

A calculation procedure of multi-digit subtraction was decided by relation between each digit number of minuend and subtrahend. In our system, 3-digit subtraction is focused because it is last step of multi-digit subtraction at elementary school in Japan. The problems and solutions are categorized 5 types show as Fig. 1. Problems become difficult in the order of number from smallest to biggest because learners learn in this order. Learners are required to solve and pose based on the 5 types in our system.

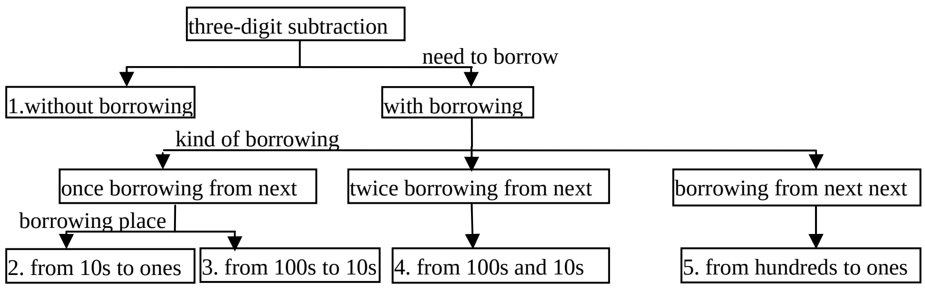

Fig. 1: Categorizing of Problem

2. Design of the ILE

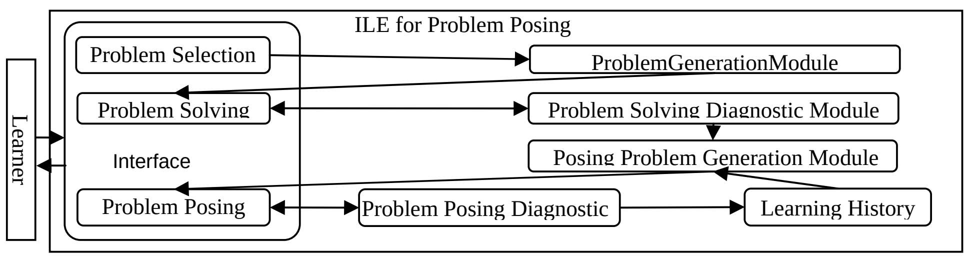

Fig. 2: Structure of the ILE

Fig 2 shows a structure diagram of our ILE which is implemented by java application.

2.1 Problem Solving

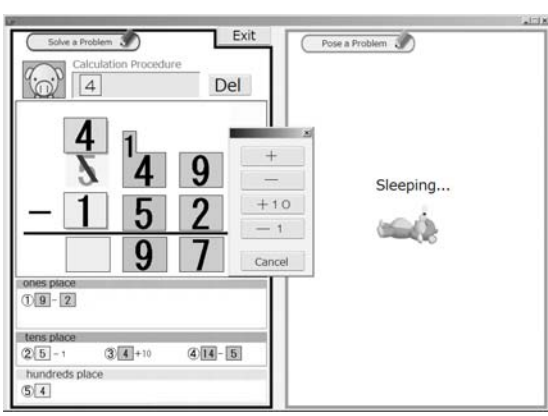

Fig. 3: Problem Solving Process

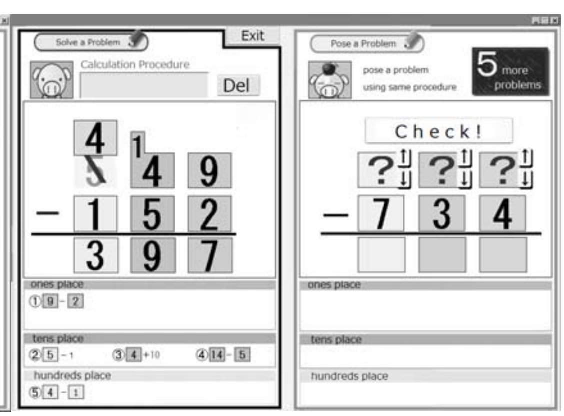

Fig. 4: Problem Posing Process

Problem Solving Interface is shown in left part of Fig. 3 This interface requires learners to solve a problem and to recognize calculation procedure clearly by interaction to the ILE. The procedure in multi-digit subtraction composed of actions which are subtracting numbers, increasing by 10 with borrowing and decreasing by 1 with borrowing. In the ILE, learners can use these action and adding numbers. These modeling of calculation procedure in multi-digit subtraction are based on research for buggy [6].

2.2 Problem Posing

Problem Posing Interface shown in right part of Fig. 4 requires learners to pose a problem which requires a same calculation procedure they solved. Posing Problem Generation Module generates numbers at random and changes some of the numbers to blank space. The blanks are filled by clicking upper arrow or downer arrow. They are required to pose six times for one type problem. In the problem posing, there two type of difficulty. Another one is type of solution type shown in Fig. 1. The difficulty of types of solution increases as number of type increase because a learner must consider more elements as number of type increase and they appear in this order in textbooks. In our ILE, however, now learners can select type of problem, so the difficulty about type isn't controlled. Another one is number of blank spaces of one type. Blank spaces are increased by learning history is decided by number they posed same type correctly. The difficulty about blank space increases this order of the steps in our ILE. The Learning History about Difficulty Step which a learner achieved is recorded. The ILE referrers the history and generates posing problem. All problems which learners can pose are 120 (5 types of problem * 6 times for one type of problem * 4 Difficulty Step).

Blank spaces (BS) one or two -> Difficulty Step 1
BS two or three -> Difficulty Step 2
BS four or five -> Difficulty Step 3
BS five or six -> Difficulty Step 4

Problem Posing Diagnostic Module specifies types of solved problem and posed problem based on the category shown in Fig. 1. If both problems are belong same type, the posed problem is correct. If types of problems are different, the posed problem is incorrect. Messages are provides about differences by level in Fig. 1. The detail of different part is also represented by comparing solution for solved problem to solution for posed problem. If solved problem belongs type 4 but a learner poses a problem belongs type 2. The ILE provides a message that tens place is different. Next, the ILE provides a message that is same at the view point of checking for borrowing but different at the view point of kind of borrowing under level of Fig. 1. Calculation procedure of posed problem is also represented beside calculation procedure of solved problem and backgrounds of different place are colored for representation about detail of different place. Development of the ILE is also considered proper because it is evaluated by opinions of some elementary school teachers.

3. Experimental Use

Our ILE was used experimentally in elementary school. Purpose of the experimental use is to confirm that our system is useful tool for solution-based problem-posing in multi-digit subtraction. In multi-digit subtraction, learning by problem-posing is difficult without support of computer because of difficult solution structure, thus our system is considered of value if learners can pose problems in multi-digit subtraction and accept our system.
132 students and 5 teachers including class teachers used our system. The students were third-grade level and just finished learning for three-digit subtraction. The teachers used our ILE before students used in arithmetic classes, and they evaluated that our system can be used at the third-grade level. In the arithmetic classes, we explained our ILE for 10 minutes at first. Next, students learned on our ILE for 50 minutes. In the learning, each learner used his own computer. Finally, we asked them and the teachers several questions.

3.1 Result of Using our ILE

Each student posed 122 problems on an average for 50 minutes. There were 93 correct problems and correct rate was 76%. Table 1 shows amount of posed problems, amount of correct problems and rate of them on each student per 10 minutes. The result shows that students continuously posed problems on our ILE for 50 minutes. Table 2 shows a relationship between posed problems, students and correct rate. The result shows that students can pose a problem at least per minute.

Table 1: Amount of posed problems and correct problems a person per 10 minutes (Column after 40 minutes includes the use after 50 minutes in circumstances of execution)

Time	0-10	10-20	20-30	30-40	40-	All
Posed problems	25.8	26.4	21.4	22.3	25.7	122
Correct problems	19.1	20.9	16.8	16.8	19.4	93
Correct rate	74%	79%	78%	75%	76%	76%

Table 2: Distribution of correct problems rate of total posed problems a student

Posed Problems	60-79	80-99	100-119	120-139	140-159	160-
Students	11	26	56	29	6	4
Correct rate	63%	71%	86%	77%	66%	42%

From the results, two concerns are occurred. One is (A) that they could pose problems without careful consideration. Another one is (B) why they posed for such a long time continuously. About (A), if they posed problems without consideration, they could not pose correct problems on this high rate. On our ILE, easiest problem posing is filling only one blank on problem type 1 in Fig. 1. Even if they pose this easiest problem, they cannot pose correct problem 50% of the time without consideration because filling number is required to be smaller or bigger than other number on same place. The rate of correct problems, however, is 76%. Therefore we guess that they considered solution structure and conducted posing problems activities. About (B), I think there are several reasons. It may be interesting for them to pose problems, to learn on our ILE, and so on. We cannot specify which element is most important, but the result about (B) is caused by interaction to actual teachers at elementary school when we developed our ILE. From the results, however, one feature is also occurred. We should discuss that (C) rate of correct problems doesn't increase with time though rate increases with time by learning usually. I guess that this feature is occurred by feature of difficulty on our ILE. Difficulty of posing problems increases with time our ILE. Amount of blank spaces increases gradually when they posed the same type of problems, and they are easy to select easy type of problem by bias. Actually, 89% students selected type 1 at first and 65% selected from type 1 to type 5 in order. Though the two difficulties increased on our ILE with time for 50 minutes, the rate of correct problems didn't decrease. There is possibility that their abilities of posing problems are improved.

Table3: Correct problems rate of each type and each difficulty
(Line: Type of Problems, Dif: Difficulty step, Num: Amount of students that even achieved difficulty 4 of the type)

	Dif. 1	Dif. 2	Dif. 3	Dif. 4	Num
Type 1	90%	94%	95%	96%	97
Type 2	77%	85%	87%	91%	84
Type 3	87%	92%	93%	90%	68
Type 4	87%	87%	83%	89%	60
Type 5	77%	80%	80%	80%	54

Table 3 shows relationship between the rate and the two difficulties that are difficulty step (is amount of blank space) in 2.2 and difficulty of type of problems. Time line is secured in each problem type because the rate in Table 3 is counted about only students who finished difficulty step 4. The rates are the worst value in difficulty step 1 in each type of problem though difficulty step 1 is the easiest. The abilities of students may be improved in posing problem of each type from the result.

3.2 Result of Questionnaires

Table 4 shows students' answers of questionnaires after they finished learning on our ILE. (s-2) and (s-3) show that over 80% (110/134=0.82, 110/134=0.82) of them felt importance of posing problem and considering solution structure (calculation procedure). (s-1) show that over 70% (94/134=0.70) of them felt that provided feedbacks from our ILE were useful. Table 5 shows teachers' answers of one of questionnaires. All teachers answered that our ILE was useful for the class. There are also some comments such as "Though I think it is impossible for children who are not a good scholar, it is enough useful by using more than once", "It is useful as brush up on senior children at elementary school or use in introductory lecture depending on usage situation" in comment of free description.

Table 4: An evaluation of the students

	Yes	So-So	No
(s-1)Were feedbacks helpful?	94	19	21
(s-2)Did you consider calculation procedure?	110	22	2
(s-3)Do you think it is important to consider it	110	21	3

Table 5: Our ILE is useful for the class? (an evaluation of the 5 teachers)

I think that it is useful as it is.	2
I think that it is useful if it is improved a little.	3
I think that it is useful if it is improved greatly	0
I don't think that it is useful.	0
I don't know	0

4. Conclusion

In this paper, we described developing and evaluation of a learning environment for *Solution-based Problem-Posing* in the case of multi-digit subtraction and an experimental use at elementary school. We realize it with the method that learners solve a problem at first and pose problem which has same solution because it is difficult to provide solution of multi-digit subtraction in terms. The results of experimental use show that students can learn by posing problems on our ILE and that students and teachers accept the activity and our ILE. Though the learning effect isn't confirmed yet, the activity can be not realized without our ILE. Thus, it has meaning. It is our futures work to research a quantitative evaluation of the learning effect using our ILE.

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